Field Topology Visualization for Space Plasmas

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Vector fields are traditionally objects of interests for visualization. They are the
mathematical language of many research and engineering areas like e. g. fundamental
physics, optics, solid mechanics, or fluid and plasma dynamics. Vector variables are in this
context velocity, vorticity, magnetic or electric fields, and a force. From a theoretical
viewpoint, vector fields have much attention form mathematicians, leading to a precise and
rigorous framework that constitutes the basis of specific visualization methods. Of
particular interest is Poincare’s work [Abraham and Shaw, 1992; Cai et al., 2001;
Guckenheim and Holmes, 1983; Wiggins et al., 2003] that laid down the foundation of a
geometric interpretation of vector fields associated to dynamical systems at the end of the
19th century: the analysis of the phase portrait provides an efficient and aesthetic way to
apprehend the information contained in abstract vector data. Nowadays, following this
theoretical inheritance, scientists typically focus their study on the topology of vector
datasets provided by large-scale numerical simulations. A typical and very active
application field is fluid and plasma dynamics where complex structural behaviors are
investigated in the light of their topology. It was shown that for instance those topological
features are directly involved in crucial aspects of flight stabilities like flow separation in
fluid dynamics and magnetic reconnection or sometimes referred to “bifurcation” in plasma
physics. Extracting and studying this structure permits to focus the analysis on essential
properties. For visualization purposes, the depiction of the topology results in synthetic
representations that transcribe the fundamental characteristics of the vector dat.
Moreover, it permits fast extraction of global flow or magnetic structures that are directly
related to features of interests in curious practical applications. In the present report, we
focus on the magnetic field topology in plasma flows. In the field topology approaches, for
example, please see [Abraham and Shaw, 1992; Cai et al., 2001; Guckenheimer and Holmes, 1983; Wiggins et al., 2003], a pattern of magnetic field lines generates the “phase portrait” of a three-dimensional vector field. Here, we briefly introduce these field topology approaches in this tutorial. Two “phase portraits” of the magnetic field have the same topology if a one-to-one mapping from one phase portrait to the other phase portrait preserves the paths that are magnetic field lines in the phase portrait [Abraham and Shaw, 1992]. We can also say in this case that two “phase portraits” are homeomorphic. Let us consider a two-dimensional case and imprint a two-dimensional phase portrait on a “sheet of a rubber” that may be deformed in any way without folding or tearing. Any deformation of this sheet rubber with its associated phase portrait is a path-preserving mapping, where a magnetic field line is preserved. It is known that all characteristics of the phase portrait that remains invariant under homeomorphic mappings are represented as topological properties. They are namely (i) the number and types of magnetic null points, i.e. where the amplitude of the magnetic field is exactly zero, and are also called magnetic nulls or neutral points, (ii) the existence of a path called a separator line connecting the magnetic null points, and (iii) the existence of closed magnetic field lines. The types of the magnetic null points can be classified by their eigenvalues, and they are always saddles in magnetic field. The topological structure of the magnetic field can be characterized by the set of topological properties of the phase portrait [Abraham and Shaw, 1992; Arnold, 1973; Cai et al., 2001; Guckenheim and Holmes, 1983; Tobak and Peake, 1982; Tricoche et al., 2002; Wiggins et al., 2003].


